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EXCITATION OF HIGHER ORDER MODES BY A STEP DISCONTINUITY OF A CIRCULAR WAVEGUIDE

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ABSTRACT

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The problem of excitation of higher order modes by a step discontinuity in a circular waveguide with an incident dominant H_{11} mode is considered. An approximate method is used to solve this problem. In this method, it is assumed that the tangential electric field at the discontinuity is zero everywhere except in the aperture where it is equal to the incident tangential electric field. Relative amplitudes of these excited modes are given and discussed.

Author

I. INTRODUCTION

The use of a horn as a radiator has been considered either theoretically or experimentally by many investigators (Ref. 1-4). However, most previous work has been concerned chiefly with the radiation characteristics of a horn radiator excited by a single waveguide mode. In order to suppress side lobes of the radiation pattern and still retain an equal beamwidth for the main lobe, Potter (Ref. 5) recently investigated the use of a horn radiator excited by a combination of two waveguide modes. He has shown that a significant reduction of the side lobe amplitude is achieved if the E_{11} circular waveguide mode is properly used in combination with the commonly used dominant H_{11} circular waveguide mode (i. e., the relative amplitude and phase of the E_{11} and H_{11} modes must be properly chosen). One of the simplest ways of generating these modes is to make an abrupt discontinuity of the cross section of the waveguide. The purpose of this report is to investigate this excitation problem.

The problem of an obstacle or discontinuity in waveguides has been treated by Schwinger (Ref. 6) and his followers (Ref. 7-9) using the integral equation and variational techniques. However, when more than one propagating mode can exist in the waveguide, the solution of the resultant integral equation becomes very involved. Hence, an approximate method is introduced to consider the present problem. In this method it is assumed that the tangential electric field at the discontinuity is a known quantity. Namely, it is the tangential electric field of the incident wave at the aperture and is otherwise zero at the discontinuity. The excited electromagnetic waves are expanded in terms of the orthonormal modes of the waveguide. The expansion coefficients are then obtained by matching the tangential electric field at the discontinuity. Relative amplitudes of the excited waves are plotted. Results will be discussed.

II. FORMULATION OF THE PROBLEM

One end of a circular waveguide of radius a, called waveguide 1, is connected with the other end of a circular waveguide of radius b, called waveguide 2, as shown in Fig. 1. It is assumed that $b > a$. Only the dominant H_{11} mode may propagate in waveguide 1; all other modes are evanescent. The only propagating modes in waveguide 2 are the H_{11}, H_{21}, H_{01}, E_{01}, and E_{11} modes. Because of the symmetrical characteristics of the discontinuity, the azimuthal dependence of all excited modes will be either sin θ or cos θ if the incident wave is the dominant H_{11} wave.

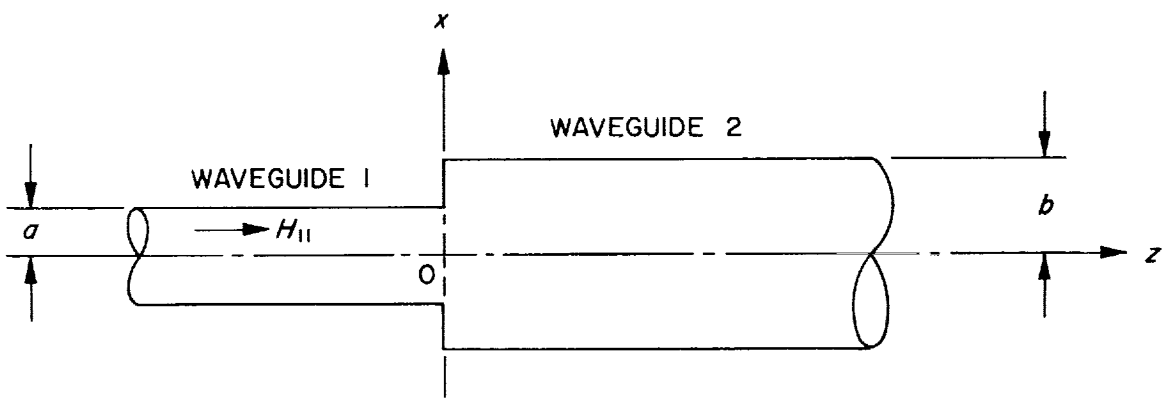

Fig. 1. Geometrical configuration

The tangential electric and longitudinal magnetic field components of the incident H_{11} wave in a circular waveguide may be given by[1]

$$H_z^{(i)} = H_0 \, \alpha_1^i \, J_1 (\gamma_{11} \rho) \cos \theta \, e^{ih_{11} z} \tag{1a}$$

$$E_\theta^{(i)} = - \frac{i\omega\mu_0}{\gamma_{11}} H_0 \, \alpha_1^{(i)} \, J_1' (\gamma_{11} \rho) \cos \theta \, e^{ih_{11} z} \tag{1b}$$

$$E_\rho^{(i)} = - \frac{i\omega\mu_0}{\gamma_{11}^2} H_0 \, \alpha_1^{(i)} \frac{1}{\rho} J_1 (\gamma_{11} \rho) \sin \theta \, e^{ih_{11} z} \tag{1c}$$

[1] The nomenclature used in this Report follows that used in Ref. 10.

where

$$\alpha_1^{(i)} = \sqrt{\frac{2}{\pi}} \frac{\gamma_{11}^2}{\sqrt{(\gamma_{11}a)^2 - 1}} \frac{1}{J_1(\gamma_{11}a)}, \quad \gamma_{11}a = 1.8412, \; h_{11}^2 = k_0^2 - \gamma_{11}^2$$

$$k_0^2 = \omega^2 \mu \epsilon_0$$

and H_0 is the amplitude of the incident wave. The cylindrical coordinates (ρ, θ, z) are used. The harmonic time dependence, $e^{-i\omega t}$, is assumed and suppressed throughout. The appropriate expression for the transverse electric field components of the transmitted wave in waveguide 2 is (Ref. 10):

$$\mathbf{E}_t^{(t)} = \sum_p A_p \mathbf{E}_{tp}^m + B_p \mathbf{E}_{tp}^e \tag{2}$$

Where

$$\mathbf{E}_{tp}^m = \alpha_p^m \left[-\frac{i\omega\mu_0}{\gamma_{1p}^m} J_1'(\gamma_{1p}^m \rho) \cos\theta \, \mathbf{e}_\theta - \frac{i\omega\mu_0}{\gamma_{1p}^{m\,2}} \frac{1}{\rho} J_1(\gamma_{1p}^m \rho) \sin\theta \, \mathbf{e}_\rho \right] e^{ih_{1p}^m z}$$

$$\mathbf{E}_{tp}^e = \alpha_p^e \left[\frac{ih_{1p}^e}{\gamma_{1p}^{e\,2}} \frac{1}{\rho} J_1(\gamma_{1p}^e \rho) \cos\theta \, \mathbf{e}_\theta + \frac{ih_{1p}^e}{\gamma_{1p}^e} J_1'(\gamma_{1p}^e \rho) \sin\theta \, \mathbf{e}_\rho \right] e^{ih_{1p}^e z},$$

with

$$h_{1p}^m = \sqrt{k^2 - \gamma_{1p}^{m\,2}}, \qquad h_{1p}^e = \sqrt{k^2 - \gamma_{1p}^{e\,2}}$$

$$J_1'(\gamma_{1p}^m b) = 0, \qquad J_1(\gamma_{1p}^e b) = 0$$

$$\alpha_p^m = \sqrt{\frac{2}{\pi}} \frac{\gamma_{1p}^{m\,2}}{\sqrt{(\gamma_{1p}^m b)^2 - 1}} \frac{1}{J_1(\gamma_{1p}^m b)}, \qquad \alpha_p^e = \sqrt{\frac{2}{\pi}} \frac{\gamma_{1p}^e}{b} \frac{1}{J_2(\gamma_{1p}^e b)}$$

where the prime signifies the derivative of the function with respect to its argument; $\mathbf{e}_\theta$ and $\mathbf{e}_\rho$ are respectively the unit vector in θ and ρ directions. A_p and B_p are yet unknown coefficients. They may be found by matching the fields at the discontinuity.

III. MATCHING AT THE DISCONTINUITY

At the discontinuity, i.e., at $z = 0$, the transverse components of the transmitted electric field must be identical with the aperture electric field $\mathbf{E}_{ap}$ in the aperture and must be zero on the perfectly conducting obstacle, i.e.,

$$\begin{aligned} \mathbf{E}_t^{(t)} &= \mathbf{E}_{ap} \qquad \text{for } 0 \leq \rho \leq a \\ &= 0 \qquad \text{for } a \leq \rho \leq b \end{aligned} \tag{3}$$

at $z = 0$. $\mathbf{E}_t^{(t)}$ is given by Eq. (2) with $z = 0$.

Applying the orthogonality relations (Ref. 10)

$$\int_0^{2\pi}\int_0^b (\mathbf{E}_{tp}^e \cdot \mathbf{E}_{tq}^m)\,\rho\,d\rho\,d\theta = 0 \qquad \text{for all } p \text{ and } q \tag{4}$$

$$\begin{aligned} \int_0^{2\pi}\int_0^b (\mathbf{E}_{tp}^e \cdot \mathbf{E}_{tq}^e)\,\rho\,d\rho\,d\theta &= 0 \qquad \text{for } p \neq q \\ &= h_{1p}^{e\,2} \qquad \text{for } p = q \end{aligned} \tag{5}$$

$$\begin{aligned} \int_0^{2\pi}\int_0^b (\mathbf{E}_{tp}^m \cdot \mathbf{E}_{tq}^m)\,\rho\,d\rho\,d\theta &= 0 \qquad \text{for } p \neq q \\ &= \omega^2\mu_0^2 \qquad \text{for } p = q \end{aligned} \tag{6}$$

to Eq. (3), one obtains

$$A_p = \frac{1}{\omega^2\mu^2}\int_0^{2\pi}\int_0^a (\mathbf{E}_{ap} \cdot \mathbf{E}_{tp}^m)\,\rho\,d\rho\,d\theta \tag{7}$$

$$B_p = \frac{1}{h_{1p}^{e\,2}}\int_0^{2\pi}\int_0^a (\mathbf{E}_{ap} \cdot \mathbf{E}_{tp}^e)\,\rho\,d\rho\,d\theta \tag{8}$$

It is noted that the tangential aperture electric field $\mathbf{E}_{ap}$ is still an unknown quantity. If one follows the Schwinger (Ref. 6) formulation, one then obtains a Schwinger-type integral equation in which $\mathbf{E}_{ap}$ is the unknown function. The solution for $\mathbf{E}_{ap}$ is very involved and tedious if more than one propagating mode is allowed in waveguide 2 as in the present case. To overcome this difficulty, we shall assume that the tangential aperture electric field $\mathbf{E}_{ap}$ is identical with the tangential components of the incident electric field at $z = 0$ (see Eq. 1), as a first-order approximation. In other words,

$$\mathbf{E}_{ap} \approx H_0\, a_1^{(i)} \left[-\frac{i\omega\mu_0}{\gamma_{11}} J_1'(\gamma_{11}\rho) \cos\theta\, \mathbf{e}_\theta - \frac{i\omega\mu_0}{\gamma_{11}^2} \frac{1}{\rho} J_1(\gamma_{11}\rho) \sin\theta\, \mathbf{e}_\rho \right] \tag{9}$$

The limitations of this approximation are similar to the ones made for the slot antenna (Ref. 11). A discussion of slots in waveguides has been given by Stevenson (Ref. 12). Substituting Eq. (9) into Eq. (7) and (8), and carrying out the integration gives

$$A_p = H_0 \frac{2(\gamma_{11} a)\, \gamma_{1p}^{m\,2}}{(\gamma_{11}^2 - \gamma_{1p}^{m\,2})} \cdot \frac{1}{\sqrt{[(\gamma_{11} a)^2 - 1]\,[(\gamma_{1p}^m b)^2 - 1]}} \frac{J_1(\gamma_{1p}^m a)}{J_1(\gamma_{1p}^m b)} \tag{10}$$

$$B_p = -H_0 \frac{\omega\mu_0\, \gamma_{11}^2}{h_{1p}^e (\gamma_{11} b) \sqrt{(\gamma_{11} a)^2 - 1}} \frac{X}{(\gamma_{11}^2 - \gamma_{1p}^{e2})\, J_1(\gamma_{11} a)\, J_2(\gamma_{1p}^e b)} \tag{11}$$

with

$$\begin{aligned} X = {} & \gamma_{11} a \,[J_2(\gamma_{1p}^e a)\, J_2'(\gamma_{11} a) - J_0(\gamma_{1p}^e a)\, J_0'(\gamma_{11} a)] \\ & + \gamma_{1p}^e a \,[J_0(\gamma_{11} a)\, J_0'(\gamma_{1p}^e a) - J_2(\gamma_{11} a)\, J_2'(\gamma_{1p}^e a)] \end{aligned} \tag{12}$$

One obtains the complete expressions for the transverse components of the transmitted electric field by substituting the expressions for A_p and B_p into Eq. (2):

$$E_\rho^{(t)} = -\frac{i\omega\mu_0}{a} H_0 \, 2\sqrt{\frac{2}{\pi}} \, C \sum_p \left[P_p \frac{1}{\gamma_{1p}^m \rho} J_1 (\gamma_{1p}^m \rho) \sin\theta \, e^{ih_{1p}^m z} + Q_p J_1' (\gamma_{1p}^e \rho) \sin\theta \, e^{ih_{1p}^e z} \right] \tag{13}$$

$$E_\theta^{(t)} = -\frac{i\omega\mu_0}{a} H_0 \, 2\sqrt{\frac{2}{\pi}} \, C \sum_p \left[P_p J_1' (\gamma_{1p}^m \rho) \cos\theta \, e^{ih_{1p}^m z} + Q_p \frac{1}{\gamma_{1p}^e \rho} J_1 (\gamma_{1p}^e \rho) \cos\theta \, e^{ih_{1p}^e z} \right] \tag{14}$$

where

$$C = \frac{\gamma_{11} a}{\sqrt{(\gamma_{11} a)^2 - 1}} \; \frac{\gamma_{11}^{m^3} a^3}{(\gamma_{11}^2 a^2 + \gamma_{11}^{m^2} a^2) \, [(\gamma_{11}^m b)^2 - 1]} \; \frac{J_1 (\gamma_{11}^m a)}{J_1 (\gamma_{11}^m b)} \tag{15}$$

$$P_p = \left(\frac{\gamma_{11}^2 a^2 - \gamma_{11}^{m^2} a^2}{\gamma_{11}^2 a^2 - \gamma_{1p}^{m^2} a^2} \right) \frac{\sqrt{(\gamma_{11}^m b)^2 - 1}}{\sqrt{(\gamma_{1p}^m b)^2 - 1}} \; \frac{J_1 (\gamma_{1p}^m a)}{J_1 (\gamma_{11}^m a)} \; \frac{J_1 (\gamma_{11}^m b)}{J_1 (\gamma_{1p}^m b)} \; \frac{\gamma_{1p}^{m^3} a^3}{\gamma_{11}^{m^3} a^3} \; \frac{\sqrt{(\gamma_{11}^m b)^2 - 1}}{\sqrt{(\gamma_{1p}^m b)^2 - 1}} \tag{16}$$

$$Q_p = \frac{1}{2} \; \frac{(\gamma_{11}^2 a^2 - \gamma_{11}^{m^2} a^2) \, [(\gamma_{11}^m b)^2 - 1]}{\gamma_{11}^{m^3} a b^2} \; \frac{J_1 (\gamma_{11}^m b)}{J_1 (\gamma_{11}^m a)} \; \frac{X}{(\gamma_{11}^2 a^2 - \gamma_{1p}^{e^2} a^2) \, J_1 (\gamma_{11} a) \, J_2^2 (\gamma_{1p}^e b)} \tag{17}$$

and X is given by Eq. (12)

Numerical computations of C, P_p, and Q_p were carried out using the IBM 7090 computer for various values of $k_0 a$ and $k_0 b$. As mention in Sec. II, we assume that the size of waveguide 1 is such that only the dominant H_{11} mode may propagate and that the size of waveguide 2 is such that only H_{11} and E_{11} modes may propagate.[2] Hence, the range of $k_0\, a$ and $k_0\, b$, that we will consider, are respectively $1.841 \leq k_0\, a \leq 5.331$ and $1.841 \leq k_0\, b \leq 5.331$. P_p and Q_p for various values of $k_0\, a$ and $k_0\, b$ are plotted in Fig. 2 through 5. It can be seen that, as expected, if $k_0\, b \approx k_0\, a$, the only mode that is strongly excited is the H_{11} wave. As the difference between $k_0\, a$ and $k_0\, b$ becomes larger, other modes may be more strongly excited. It is also interesting to note that for some specific values of $k_0\, a$, the amplitudes of certain higher order modes are zero. It should be recalled that the analysis given here is an approximate one. Hence, the results are applicable only when the assumption, that the tangential aperture electric field is the incident electric field at the discontinuity, may be used.

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[2] Because of the symmetrical nature of the discontinuity, H_{01}, E_{01}, and H_{21} propagating modes in waveguide 2 are not excited.

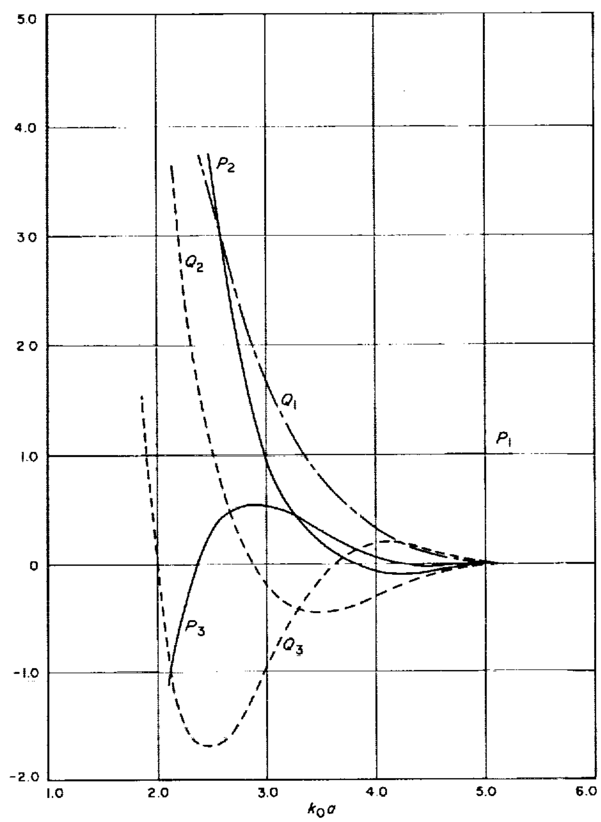

Fig. 2. Relative amplitudes of excited modes for $k_0\, b = 5.251$

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The author wishes to thank Mr. Philip D. Potter of the Jet Propulsion Laboratory, Pasadena, California, for many valuable discussions.

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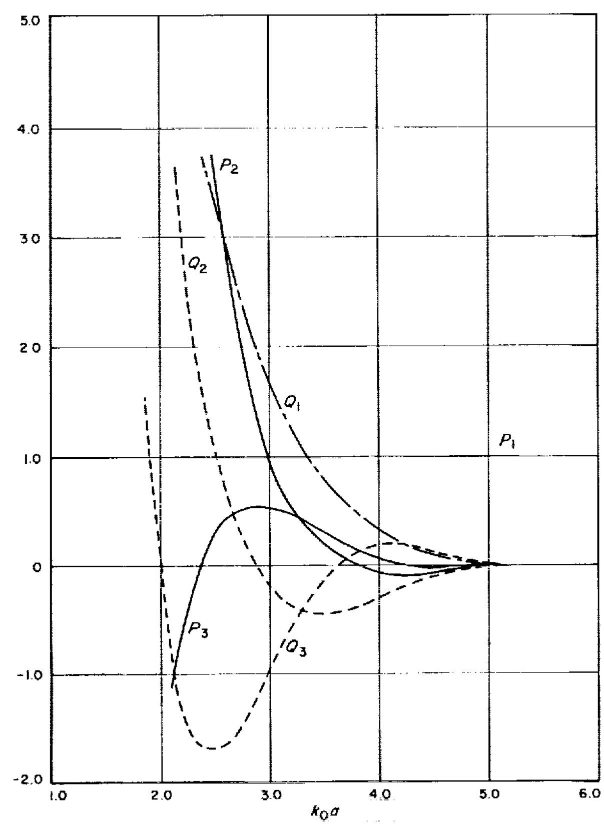

Fig. 2. Relative amplitudes of excited modes for $k_0\,b = 5.251$

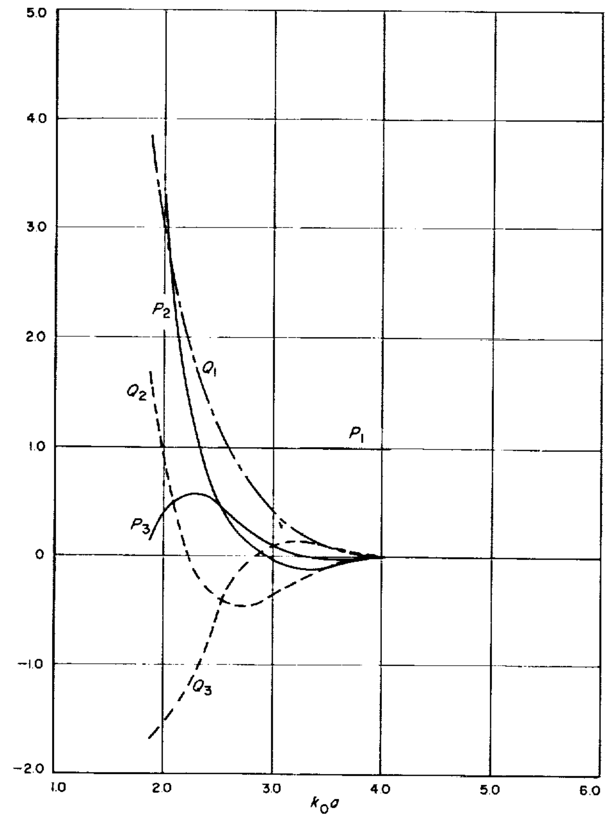

Fig. 3. Relative amplitudes of excited modes for $k_0\, b = 4.051$

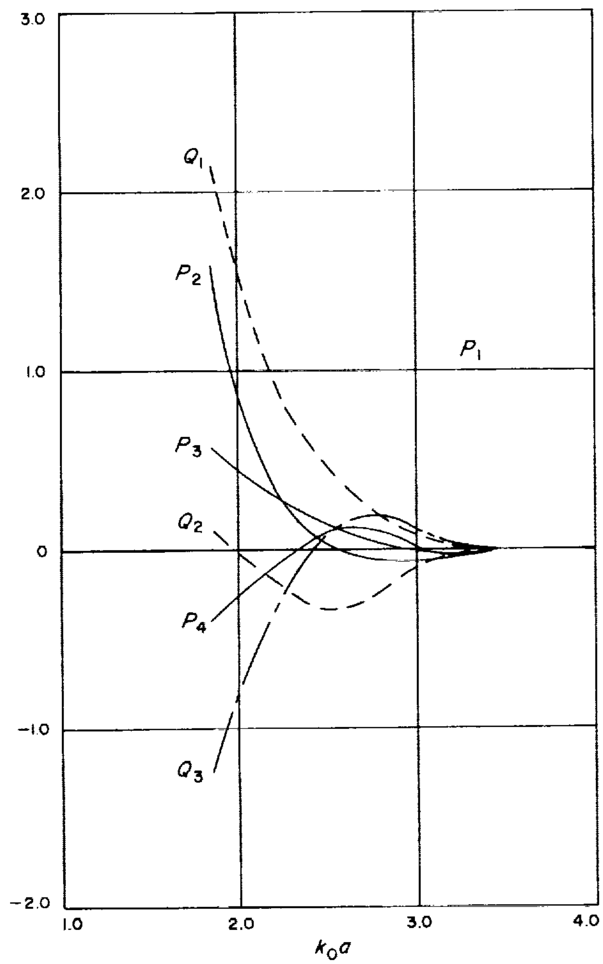

Fig. 4. Relative amplitudes of excited modes for $k_0 b = 3.451$

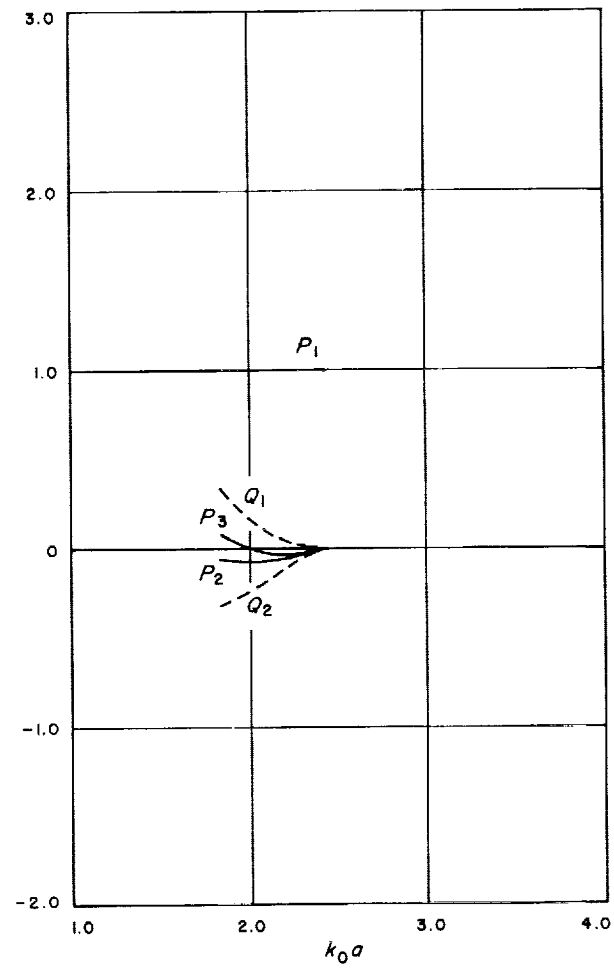

Fig. 5. Relative amplitudes of excited modes for $k_0 b = 2.451$

ACKNOWLEDGEMENT

The author wishes to thank Mr. Philip D. Potter of the Jet Propulsion Laboratory, Pasadena, California, for many valuable discussions.

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